**Definition.** Consider a directed acyclic graph G = (V, E) and two nodes u and v such that there does not exist a path from node v to node u. *Path addition* is the operation in which a new path from node u to node v is added. Let l denote the length of the new path. If l = 1, an edge (u, v) is added. If l > 1, nodes w\_1, \ldots, w\_{l - 1} and edges (u, w\_1), (w\_1, w\_2), \ldots, (w\_{l - 2}, w\_{l - 1}), (w\_{l - 1}, v) are added.

**Theorem.** Consider a directed acyclic graph G = (V, E), a source node s, and a target node t. In addition, consider two nodes u and v such that there does not exist a path from node v to node u. Adding a path from node u to node v increases the intermediacy \phi\_w of any node w located on a path from node s to node u or from node v to node t.

**Definition.** Consider a directed acyclic graph G = (V, E) and two nodes u and v such that there exists at least one path from node u to node v. Let P\_{uv} \subseteq V denote the set of all nodes located on a path from node u to node v, including nodes u and v themselves. *Path contraction* is the operation in which all nodes in P\_{uv} are contracted. This means that the nodes in P\_{uv} are replaced by a new node w. Edges connecting two nodes that are both in P\_{uv} are removed. If an edge points from a node v’ \notin P\_{uv} to a node in P\_{uv}, the edge is replaced by a new edge (v’, w). If an edge points from a node in P\_{uv} to a node v’ \notin P\_{uv}, the edge is replaced by a new edge (w, v’).

**Theorem.** Consider a directed acyclic graph G = (V, E), a source node s, and a target node t. In addition, consider two nodes u and v such that there exists at least one path from node u to node v and such that any source-target path that goes through node u or node v also goes through the other node. Contracting paths from node u to node v increases the intermediacy \phi\_w of any node w located on a path from node s to node u or from node v to node t.